

$X(3872)$ as a 1D_2 charmonium state

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The 1D_2 charmonium assignment for the $X(3872)$ meson is considered, as prompted by a recent result from the BABAR Collaboration, favouring 2^{-+} quantum numbers for the X . It is shown that established properties of the $X(3872)$ are in a drastic conflict with the 1D_2 $c\bar{c}$ assignment.

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Seven years after its discovery, the $X(3872)$ meson has confirmed once again its reputation of *enfant terrible* of meson spectroscopy. The state was first seen by Belle [1] in the $\pi^+\pi^-J/\psi$ mode and then confirmed, in the same discovery mode, by the CDF [2], DØ [3], and BABAR [4] Collaborations. According to the CDF analysis of the dipion mass spectrum and the angular distribution in the $\pi^+\pi^-J/\psi$ mode [5], only 1^{++} and 2^{-+} assignments are able to describe the data. Then, while the nature of the state remains controversial, there are good phenomenological reasons to assign it 1^{++} quantum numbers.

First, the X resides at the $D^0\bar{D}^{*0}$ threshold, which prompts a considerable admixture of a molecule in its wave function. Furthermore, CDF concludes that the $\pi^+\pi^-$ come from the ρ [6] which, together with the Belle observation of the $\omega J/\psi$ mode [7], points to a considerable isospin violation. The latter can be explained naturally in the molecular model of the X , which implies 1^{++} quantum numbers. In addition, the X was also observed in the $D^0\bar{D}^{*0}$ mode with a significant rate [8–10]. Both $\rho J/\psi$ and $D^0\bar{D}^{*0}$ modes were analysed simultaneously in Refs. [11–13], and it was shown that indeed the data were compatible with a large admixture of the $D^0\bar{D}^{*0}$ molecular component in the wave function of the X .

However, a recent analysis of the decay $B \rightarrow K\omega J/\psi$ data performed by the BABAR Collaboration [14] indicates that inclusion of an extra unit of the orbital angular momentum in the $\omega J/\psi$ system improves significantly the overall description of the observed $\pi^+\pi^-\pi^0$ mass distribution, which implies a negative P -parity of the $X(3872)$ state. Although this new BABAR result is fully compatible with the 2^{-+} assignment allowed by CDF, if confirmed, it clearly challenges our understanding of the charmonium spectroscopy above the open-charm threshold. Here we investigate the most conventional explanation for the 2^{-+} $X(3872)$ as the 1D_2 charmonium state.

In case of charmonium D -levels we have an experimental anchor at our disposal — the $\psi(3770)$ vector state which is dominantly a $c\bar{c}$ state, with the angular momentum of the quark–antiquark pair $L = 2$ and the total quark spin $S = 1$ (1D_2 state has $L = 2$ and $S = 0$). As c -quark is heavy, the spin–orbit force, which splits spin-triplet and spin-singlet levels, is not large and all D -levels are degenerate in the leading-order approximation. Hence, one may use the data on the 3D_1 level to estimate the mass and matrix elements of the 1D_2 level.

It became clear long ago that the 1D_2 assignment for the X disagreed with quark model mass estimates (see, for example, Refs. [15–17]). Indeed, quark models usually predict the 1D_2 mass in the range $3770 \div 3830$ MeV, while the mass difference between the 1D_2 and 3D_1 levels is predicted to be, averagely, $20 \div 30$ MeV. Thus quark models cannot accommodate the $X(3872)$ as a 1D_2 state. The same conclusion was drawn in a recent paper [18]. One might think that inclusion of various D -meson loops changes this statement. It is not the case, however. Loop calculations in the Cornell decay model [19] and in the 3P_0 decay model [20] give for the mass of the 1D_2 level 3838 MeV and 3800 MeV, respectively.

The arguments based solely on the mass calculations are, of course, not enough to rule out the charmonium assignment for the X . However further reasons for the 1D_2 interpretation to be problematic have started to show up. First, radiative decay transitions $^1D_2 \rightarrow \gamma J/\psi(\psi')$ rates are shown to be incompatible with the data [21]. Second, the production cross section of the 1D_2 level at CDF is predicted to be much smaller than the one actually observed for the X [18]. In this paper we identify a couple of new problems. Namely, we expand on the issue of radiative decays and discuss the $D^0\bar{D}^{*0}$ mode of the X .

The BABAR Collaboration has reported the following rates for the decays $X(3872) \rightarrow \gamma J/\psi(\psi'(3686))$ [22]:

$$\begin{aligned} \mathcal{B}_1 &= Br(B^\pm \rightarrow K^\pm X) Br(X \rightarrow \gamma J/\psi) \\ &= (2.8 \pm 0.8 \pm 0.2) \times 10^{-6}, \\ \mathcal{B}_2 &= Br(B^\pm \rightarrow K^\pm X) Br(X \rightarrow \gamma \psi') \\ &= (9.5 \pm 2.9 \pm 0.6) \times 10^{-6}. \end{aligned} \quad (1)$$

In the meantime, the upper limit on the total branching fraction $B \rightarrow K X(3872)$ imposed by BABAR [23] is

$$\mathcal{B}_{tot} = Br(B \rightarrow K X) < 3.2 \times 10^{-4}. \quad (2)$$

Below we demonstrate that measurements (1) and (2) cannot be reconciled with each other under the assumption of the X being a 1D_2 charmonium. To this end we notice that the leading multipole for the $^1D_2 \rightarrow \gamma V$ (V is a vector charmonium) transition is $M1$, with the width given by a standard formula (see, for example, Ref. [17]):

$$\begin{aligned} \Gamma(^{2S+1}L_J \rightarrow ^{2S'+1}L'_{J'}) & \\ &= \frac{4}{3} \frac{2J'+1}{2L+1} \delta_{LL'} \delta_{S,S'+\pm 1} \frac{\alpha e_c^2}{m_c^2} |\langle \psi_f | \psi_i \rangle|^2 E_\gamma^3, \end{aligned} \quad (3)$$

where m_c is the charmed quark mass, $e_c = 2/3$, $\psi_i(\psi_f)$ is the initial(final)-state radial wave function, and E_γ is the photon energy. In this formalism, the transition $^1D_2 \rightarrow \gamma J/\psi(\psi')$ is a so-called hindered transition, so that $\langle \psi_f | \psi_i \rangle = \sin \theta$, where θ is the $^3S_1 - ^3D_1$ mixing angle. Thus the amplitude simply vanishes if $J/\psi(\psi')$ is assumed to be a pure 3S_1 state. The standard value for the ψ' is $\theta \approx 12^\circ$, which gives (for $m_c = 1.5$ GeV):

$$\Gamma(^1D_2(3872) \rightarrow \gamma\psi') \approx 6.6[\text{keV}] \sin^2 \theta \approx 0.29 \text{ keV}. \quad (4)$$

Notice that, being almost a pure 3S_1 state, J/ψ possesses a tiny mixing angle θ , so that even a much larger photon energy ($E_\gamma = 698$ MeV for the $\gamma J/\psi$ final state versus $E_\gamma = 186$ MeV for $\gamma\psi'$) cannot provide a sizeable contribution of this, formally leading, $M1$ transition. Therefore, contributions of higher multipoles have to be considered. In Ref. [21] both widths were calculated in a quite elaborated (though rather model-dependent) NRQCD approach, with the result:

$$\Gamma(^1D_2(3872) \rightarrow \gamma\psi') \approx 0.45 \div 0.5 \text{ keV}, \quad (5)$$

$$\Gamma(^1D_2(3872) \rightarrow \gamma J/\psi) \approx 6.8 \div 9.5 \text{ keV}, \quad (6)$$

claimed in Ref. [21] to contradict the BABAR data (1).

For the case of the ψ' , the $M1$ contribution from the $^3S_1 - ^3D_1$ mixing to the result (5) is indeed dominant and it is in a good agreement with the simple estimate (4). Notice that formula (3) does not take into account recoil corrections, while the formalism of Ref. [21] accounts for the recoil only via the multipole expansion. Because of a small photon energy this seems reasonable for the $\psi'(3686)$ final state while, for the J/ψ final state, the photon energy is much larger, so that the value (6) is probably an overestimation.

In order to estimate the total width of the 1D_2 charmonium we notice that, as is well-known (see, for example, Refs. [15, 17, 24]), the main radiative transition of the 1D_2 state is $^1D_2 \rightarrow \gamma^1P_1(3525)$ with the width:

$$\Gamma(^1D_2(3872) \rightarrow \gamma^1P_1(3525)) \approx 460(345) \text{ keV} [15]([24]). \quad (7)$$

Alternatively, this width can be estimated from the measured branching fractions for the transitions [25]:

$$Br(\psi(3770) \rightarrow \gamma^3P_0(3415)) = (7.3 \pm 0.9) \times 10^{-3},$$

$$Br(\psi(3770) \rightarrow \gamma^3P_1(3510)) = (2.9 \pm 0.6) \times 10^{-3}.$$

Indeed, the leading multipole is $E1$, with the width:

$$\Gamma(2^{S+1}L_J \rightarrow 2^{S'+1}L'_{J'}) = \frac{4}{3}C_{fi}\delta_{SS'}e_c^2\alpha|\langle\psi_f|r|\psi_i\rangle|^2E_\gamma^3,$$

where $C_{fi} = \max(L, L')(2J' + 1) \begin{Bmatrix} L' & J' & S \\ J & L & 1 \end{Bmatrix}^2$. In the heavy-quark limit, the dipole matrix element $\langle\psi_f|r|\psi_i\rangle$ is the same for all $D \rightarrow P$ transitions. This gives

$$\Gamma(^1D_2(3872) \rightarrow \gamma^1P_1(3525)) \approx 340 \div 440 \text{ keV}, \quad (8)$$

in a good agreement with Eq. (7).

Hadronic modes of the 1D_2 charmonium were estimated in Ref. [15]. These are the light hadrons (“ gg ”) modes and the $\eta_c\pi\pi$ mode:

$$\Gamma(^1D_2(3872) \rightarrow \text{light hadrons}) \approx 190 \text{ keV},$$

$$\Gamma(^1D_2(3872) \rightarrow \eta_c\pi\pi) \approx 210 \pm 110 \text{ keV}.$$

Together with the radiative decay modes these give for the total width the value $\Gamma_{tot} \approx 800$ keV. In principle, Γ_{tot} should also include a contribution of the $D\bar{D}^*$ modes. However, as will be shown below, with the suppression of the $D^0\bar{D}^{*0}$ mode, this contribution is negligible.

Therefore, if we use, in accordance with the Ref. [21] calculation, the value of about 8 keV for the $\gamma J/\psi$ width, we get:

$$Br(B \rightarrow KX) = (800/8) \times \mathcal{B}_1 = (2 \div 3) \times 10^{-4}, \quad (9)$$

which is compatible with Eq. (2). In the meantime, a similar estimate for the decay $X \rightarrow \gamma\psi'$ gives:

$$Br(B \rightarrow KX) = (800/0.5) \times \mathcal{B}_2 = 16 \times 10^{-3}, \quad (10)$$

where $\Gamma(X \rightarrow \gamma\psi') = 0.5$ keV as per (5) was used. This value is awfully larger than the upper limit (2). To reconcile \mathcal{B}_2 with \mathcal{B}_{tot} one needs to decrease Γ_{tot} fifty times.

We conclude in such a way that the data on the radiative decays of the $X(3872)$ do not allow for its 1D_2 charmonium interpretation, if the BABAR result on the $\gamma\psi'$ mode holds true. Notice, however, that the most recent Belle results [26] read:

$$\begin{aligned} Br(B^\pm \rightarrow K^\pm X) Br(X \rightarrow \gamma J/\psi) \\ = (1.78^{+0.48}_{-0.44} \pm 0.12) \times 10^{-6}, \end{aligned}$$

$$Br(B^\pm \rightarrow K^\pm X) Br(X \rightarrow \gamma\psi') < 3.4 \times 10^{-6},$$

which suggests that the BABAR and Belle measurements for the $\gamma\psi'$ mode contradict each other.

Let us now consider the $D^0\bar{D}^0\pi^0$ mode. In 2006 the Belle Collaboration reported an enhancement of the $D^0\bar{D}^0\pi^0$ signal observed in the reaction $B^+ \rightarrow K^+D^0\bar{D}^0\pi^0$ just above the $D^0\bar{D}^{*0}$ threshold [8], at $M_X = 3875.2 \pm 0.7^{+0.3}_{-1.6} \pm 0.8$ MeV, with the branching

$$Br(B^+ \rightarrow K^+D^0\bar{D}^0\pi^0) = (1.02 \pm 0.31^{+0.21}_{-0.29}) \times 10^{-4}.$$

The peak was confirmed by the BABAR Collaboration as well [9]. However, recently the Belle Collaboration announced a new analysis for the $D^{*0}\bar{D}^0$ case [10], and a lower peak position was obtained than reported before, namely, $M_X = 3872.9^{+0.6+0.4}_{-0.4-0.5}$ MeV, with the branching

$$Br(B^+ \rightarrow K^+D^0\bar{D}^{*0}) = (0.8 \pm 0.2 \pm 0.1) \times 10^{-4}. \quad (11)$$

This enhancement was associated with the $X(3872)$ state seen in the $D^0\bar{D}^{*0}$ mode (here and in what follows an obvious shorthand notation $D^0\bar{D}^{*0} \equiv D^0\bar{D}^{*0} + \bar{D}^0D^{*0}$ is used), and the mass shift was attributed to the proximity to the $D^0\bar{D}^{*0}$ threshold.

To proceed we find the ratio of branching fractions:

$$R = Br(B \rightarrow KD^0\bar{D}^{*0})/Br(B \rightarrow KX) > 0.25, \quad (12)$$

where the lower limit for R was deduced from the data quoted in Eqs. (2) and (11). In what follows we argue that it is not possible to reproduce such a large value of the ratio R under the assumption of the X being the 1^1D_2 charmonium. Indeed, it is claimed in Ref. [27] that the peak position in the $D\bar{D}^*$ invariant mass depends on the orbital momentum l of the $D\bar{D}^*$ pair. In particular, it is shown that with $l = 1$ it is quite easy to produce a peak at about 3 MeV above the $D\bar{D}^*$ threshold, accommodating

in such a way both BABAR [9] and old Belle [8] measurements. Depending on the model parameters, a peak much closer to the threshold can also be reproduced with $l = 1$, so that there is no contradiction with the new Belle data [10] either. The value $l = 1$ corresponds to the 2^{-+} quantum numbers of the X and therefore suggests the $1D_2$ assignment for the latter. However, then the $D^0\bar{D}^{*0}$ rate behaves as k^3 (k being the relative momentum in the $D^0\bar{D}^{*0}$ system), so the proximity to the $D^0\bar{D}^{*0}$ threshold implies a considerable suppression of the production rate. Below we make this argument quantitative.

The ratio R can be calculated as

$$R = \frac{\int_{M_-}^{M_+} dM (dBr(B \rightarrow KD^0\bar{D}^{*0})/dM)}{\int_{M_-}^{M_+} dM (dBr(B \rightarrow K\text{non}(D^0\bar{D}^{*0})/dM) + \int_{M_-}^{M_+} dM (dBr(B \rightarrow KD^0\bar{D}^{*0})/dM)}, \quad (13)$$

where the integration takes place over the mass region where the $X(3872)$ resides, conveniently defined as $M_{\pm} = M_0 \pm 10$ MeV, with M_0 being the $X(3872)$ mass. The $D^0\bar{D}^{*0}$ and $\text{non}(D^0\bar{D}^{*0})$ rates entering expression (13) are:

$$\frac{dBr(B \rightarrow KD^0\bar{D}^{*0})}{dM} = \frac{\mathcal{B}}{2\pi} \frac{g(1D_2 \rightarrow D^0\bar{D}^{*0})k^3}{(M - M_0)^2 + \Gamma_{tot}^2/4}, \quad \frac{dBr(B \rightarrow K\text{non}(D^0\bar{D}^{*0}))}{dM} = \frac{\mathcal{B}}{2\pi} \frac{\Gamma(\text{non}(D^0\bar{D}^{*0}))}{(M - M_0)^2 + \Gamma_{tot}^2/4}, \quad (14)$$

where $M_{th} = m(D^0\bar{D}^{*0})$, and the constant \mathcal{B} absorbs the details of the short-ranged dynamics of the b -quark decay. Due to the factor k^3 the expression for the $D^0\bar{D}^{*0}$ does not take a Breit-Wigner form. To account for the finite width of the D^{*0} we assume for $k(M)$ a simple ansatz [28] $k(M) = \sqrt{\mu} \sqrt{\sqrt{(M - M_{th})^2 + \Gamma_*^2/4} + (M - M_{th})}$, where μ is the $D^0\bar{D}^{*0}$ reduced mass, $\Gamma_* \approx 65$ keV is the width of the D^{*0} meson estimated from the data [25] on the $D^{*\pm}$ meson. The standard expression for the two-body relative momentum is readily reproduced as $\Gamma_* \rightarrow 0$. Finally, anticipating a strong suppression of the $D^0\bar{D}^{*0}$ mode, we substitute $\Gamma(\text{non}(D^0\bar{D}^{*0})) \approx \Gamma_{tot}$.

The coupling $g(1D_2 \rightarrow D^0\bar{D}^{*0})$ can be estimated in the $1D_2$ model for the X using the $3D_1$ state $\psi(3770)$ as a benchmark (p_{DD} is the relative $D\bar{D}$ momentum and the charged-neutral meson mass difference is neglected):

$$\Gamma(3D_1 \rightarrow D\bar{D}) = g(3D_1 \rightarrow D\bar{D})p_{DD}^3. \quad (15)$$

We now invoke the “loop theorems” proven in Ref. [29]. In particular, it is shown in this paper that, in the heavy-quark limit, strong open-flavour total widths for the states in a given $\{NL\}$ multiplet (N is the radial quantum number while L is the quark-antiquark orbital angular momentum) are equal. The heavy-quark limit implies that (i) the initial states are degenerate in mass and have the same wave functions within a given multiplet and (ii) the final two-meson states exhibit the same

degeneracy. The decay model should satisfy some general conditions listed in Ref. [29] (for example, the popular $3P_0$ pair creation model satisfies these conditions, and so does the Cornell decay model).

Specifically, in the ideal heavy-quark world, the masses of all $1D$ states are identical, and the masses of the final-state D and D^* mesons are identical too. The partial widths into certain $D^{(*)}\bar{D}^{(*)}$ channels depend on quantum numbers of a given initial state, while the sum of partial widths over all possible $D^{(*)}\bar{D}^{(*)}$ final states is the same within a given $1D$ multiplet. In the real world, if the quark-antiquark pair in the initial meson is heavy, the theorem is violated mainly by spin-dependent interactions, which remove the mass degeneracy both in the initial and final states. One may write therefore:

$$g(1D_2 \rightarrow D^0\bar{D}^{*0}) = g_0 |C(1D_2)|^2, \\ g(3D_1 \rightarrow D\bar{D}) = g_0 |C(3D_1)|^2,$$

where g_0 is the coupling constant common for all members of the $1D$ multiplet, while $C(1D_2)$ and $C(3D_1)$ are the spin-orbit recoupling coefficients for the $1D_2 \rightarrow D^0\bar{D}^{*0}$ and $3D_1 \rightarrow D\bar{D}$ decays, respectively (notice that both charged and neutral $D\bar{D}$ channels contribute to the coefficient $C(3D_1)$, while only the $D^0\bar{D}^{*0}$ channel contributes to the coefficient $C(1D_2)$). These spin-orbit recoupling coefficients were calculated in the Cornell decay model (see Table II of Ref. [19]) and in the $3P_0$ decay

TABLE I: The ratio R (see Eq. (12)) for various values of the $X(3872)$ mass M_0 and the total width Γ_{tot} .

M_0 , MeV	$\Gamma_{tot} = 200$ keV	$\Gamma_{tot} = 800$ keV	$\Gamma_{tot} = 3200$ keV
3870.8	0.023	0.023	0.022
3871.4	0.033	0.032	0.029
3872.0	0.073	0.052	0.038

model (see Table IV of Ref. [20]). Both models yield $|C(^1D_2)|^2 = \frac{3}{5} |C(^3D_1)|^2$, so that, since the $D\bar{D}$ mode is dominant for the $\psi(3770)$, we estimate the coupling $g(^1D_2 \rightarrow D^0\bar{D}^{*0})$ as:

$$g(^1D_2 \rightarrow D^0\bar{D}^{*0}) \approx \frac{3}{5} \frac{\Gamma(\psi(3770))}{p_{DD}^3}. \quad (16)$$

In Table I, we list the results for the ratio R for several values of Γ_{tot} . The masses used are $m(D^0) = 1864.84$ MeV and $m(D^{*0}) = 2006.96$ MeV. The $\psi(3770)$ width is 23 MeV. Finally, for the mass M_0 we take the same values as used in Ref. [27]. As described above, $\Gamma_{tot} = 800$ keV is our preferred value. We have also calculated the ratio R for Γ_{tot} four times smaller as well as four times larger than 800 keV, the latter value being a bit larger than 2.3 MeV quoted in PDG [25] as the upper limit for the width of the X . Clearly all values

of R listed in the Table I are far too low in comparison with the value (12) deduced from the data. In addition we confirm the $D^0\bar{D}^{*0}$ lineshapes obtained in Ref. [27], however, the rate appears to be quite small. Thus we conclude that the data on the $D^0\bar{D}^{*0}\pi^0$ mode contradict the 1D_2 charmonium interpretation of the X .

To summarize, we have shown that the 1D_2 charmonium assignment for the $X(3872)$ meson contradicts the existing data on its radiative decays and its $D^0\bar{D}^{*0}\pi^0$ mode. Our study does not challenge the 2^{-+} quantum numbers. We rather claim that, if the aforementioned experimental data are taken as a true guide, the conventional charmonium model is not able to accommodate for the 2^{-+} $X(3872)$. If the BABAR result on the quantum numbers of the $X(3872)$ persists, it would mean that some kind of a new interloper enters the game.

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- [1] S.-K. Choi *et al.* [Belle Collaboration], Phys. Rev. Lett. **91**, 262001 (2003).
 - [2] D. Acosta *et al.* [CDF II Collaboration], Phys. Rev. Lett. **93**, 072001 (2004).
 - [3] V. M. Abazov *et al.* [DØ Collaboration], Phys. Rev. Lett. **93**, 162002 (2004).
 - [4] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **71**, 071103 (2005).
 - [5] A. Abulencia *et al.* [CDF Collaboration], Phys. Rev. Lett. **98**, 132002 (2007).
 - [6] A. Abulencia *et al.* [CDF Collaboration], Phys. Rev. Lett. **96**, 102002 (2006).
 - [7] K. Abe *et al.* [Belle Collaboration], arXiv: hep-ex/0505037.
 - [8] G. Gokhroo *et al.* [Belle Collaboration], Phys. Rev. Lett. **97**, 162002 (2006).
 - [9] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **77**, 011102 (2008).
 - [10] T. Aushev *et al.* [Belle Collaboration], Phys. Rev. D **81**, 031103 (2010).
 - [11] C. Hanhart, Yu. S. Kalashnikova, A. E. Kudryavtsev, and A. V. Nefediev, Phys. Rev. D **76**, 034007 (2007).
 - [12] Yu. S. Kalashnikova and A. V. Nefediev, Phys. Rev. D **80**, 074004 (2009).
 - [13] E. Braaten and J. Stapleton, Phys. Rev. D **81**, 014019 (2010).
 - [14] P. del Amo Sanchez *et al.*, Phys. Rev. D **82**, 011101 (2010).
 - [15] T. Barnes and S. Godfrey, Phys. Rev. D **69**, 054008 (2004).
 - [16] A. M. Badalian, V. L. Morgunov, and B. L. G. Bakker, Phys. At. Nucl. **63**, 1635 (2000).
 - [17] T. Barnes, S. Godfrey, and E. S. Swanson, Phys. Rev. D **72**, 054026 (2005).
 - [18] T. J. Burns, F. Piccinini, A. D. Polosa, and C. Sabelli, Phys. Rev. D **82**, 074003 (2010).
 - [19] E. J. Eichten, K. Lane, and C. Quigg, Phys. Rev. D **69**, 094019 (2004).
 - [20] Yu. S. Kalashnikova, Phys. Rev. D **72**, 034010 (2005).
 - [21] Yu Jia, Wen-Long Sang, and Jia Xu, arXiv:1007.4541 [hep-ph].
 - [22] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **102**, 132001 (2009).
 - [23] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **96**, 052002 (2006).
 - [24] F. De Fazio, Phys. Rev. D **79**, 054015 (2009).
 - [25] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1 (2008).
 - [26] V. Bhardwaj [Belle Collaboration], belle.kek.jp/talks/QWQ2010/bhardwaj.pdf.
 - [27] W. Dunwoodie and V. Ziegler, Phys. Rev. Lett. **100**, 062006 (2008).
 - [28] M. Nauenberg and A. Pais, Phys. Rev. **126**, 360 (1962); E. Braaten and M. Lu, Phys. Rev. D **76**, 094028 (2007); C. Hanhart, Yu. S. Kalashnikova, A. V. Nefediev, Phys. Rev. D **81**, 031103 (2010).
 - [29] T. Barnes and E. S. Swanson, Phys. Rev. C **77**, 055206 (2008).